

```

/* floq.c
 * do a Floquet calculation for a single spin 1/2
 * with ZCW powder average over the whole sphere
 * Use spherical tensors as much as possible
 *
 * Basic references
 * Herzfeld and Berger, J. Chem. Phys. 73, 6021-6030 (1980)
 * Maricq and Waugh, J. Chem. Phys. 70, 3300-3316 (1978)
 * Vega, Olejniczak and Griffin, J. Chem. Phys. 80, 4832-4840 (1984)
 * Useful modern review
 * Hodgkinson and Emsley, Progress NMR Spect. 36, 201-239 (2000)
 * Our Concepts paper
 * Bain and Dumont, Concepts in Magnetic Resonance 13 159-170 (2001)
 *
 * Numerical stuff is handled by a package of routines call Meschach,
 * written in C, and available from Netlib (among other places)
 *
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 * Foundation, Inc., 59 Temple Place - Suite 330, Boston, MA 02111-1307, USA.
 */
#include <math.h>
#include <stdio.h>
#include "zmatrix2.h" /* collection of meschach headers */

#define VERSION "0200315.1"

#define SQR(x) ((x) * (x))

#define PI 3.14159265358979323846
#define TOLER 0.000001
#define NFLOQUET 17
#define NGAMMA 16
#define MAX_FIBONACCI 35
#define N_FIBONACCI 16
#define NPOINTS (8*1024)
#define START_FREQ 3000
#define DOMEQ (2*PI*2.0*START_FREQ/NPOINTS)
#define RELAX 20.0;
#define SCALE 10000.0
#define ANGLE_ERROR 0.0 /* magic = 0.955316618124509 radians*/
#define ANALYTICAL 0 /* analytical Fourier components */
#define FFT 1 /* numerical Fourier components */
#define H_AND_B 0 /* Herzfeld & Berger expressions */
#define NFOURIER 2 /* number of Fourier components for Floquet */

/* function prototypes */
void gen_freq(double alpha, double beta, int n_floquet, double theta,
             double sigma11, double sigma22, double sigma33,
             double omegar, double nuzero, VEC *freqre, VEC *freqim,
             MAT *flmat_re, MAT *flmat_im, MAT *eigvecr, MAT *eigveci, VEC *eigvals,
             ZVEC *rhozero, ZVEC *Uinv_rho);
void accum(double weight, double omegar, int n_floquet, double centreband,
           VEC *eigvals, VEC *spc, ZVEC *rhozero, ZVEC *Uinv_rho, VEC *sideb);
void mult_d2(double cbt, double sbt, ZVEC *in, ZVEC *out);
void mult_rz(double cal, double sal, ZVEC *in, ZVEC *out);
int makefibon(int *fibon);
void cg_(int nm, int n, double **ar, double **ai, double *wr, double *wi,
        int matz, double **zr, double **eigveci, double *fv1, double *fv2,
        double *fv3, int *ierr);

```

```

/*****
/***** START OF MAIN PROGRAM *****/
/*****
void main()

{
    int i, j, k;
    int n_floquet, fibon_order;

    double sigma11, sigma22, sigma33;
    double alpha, beta;
    double fac, f0, f1;
    double omegar, nuzero, weight, centreband, freq, theta;

    int fibon[MAX_FIBONACCI];

    /*
    * these are all data structures for the meschach package of
    * numerical subroutines
    */

    MAT *flmat_re, *flmat_im;
    MAT *eigvecr, *eigveci;
    VEC *eigvals;
    VEC *spc;
    VEC *sideb;
    VEC *freqre, *freqim;

    ZVEC *rhozero, *Uinv_rho;

    FILE *outp; /* will contain the spectrum */

    /* set up the parameters */
    fibon_order = N_FIBONACCI;
    n_floquet = NFLOQUET;

    /* in Herzfeld & Berger, these give mu = 13.0, rho = 0.2 */
    sigma11 = -15.34; sigma22 = -0.364; sigma33 = 22.1;
    nuzero = 2*PI*50.0;
    omegar = 2*PI*144.0;

    centreband = nuzero * (sigma11 + sigma22 + sigma33)/3.0;

    theta = acos(1.0/sqrt(3.0)) + ANGLE_ERROR; /* rotor angle */

    /* allocate and initialize some memory */
    freqre = v_get(NGAMMA);
    freqim = v_get(NGAMMA);

    flmat_re = m_get(2*n_floquet+1, 2*n_floquet+1);
    flmat_im = m_get(2*n_floquet+1, 2*n_floquet+1);
    eigvecr = m_get(2*n_floquet+1, 2*n_floquet+1);
    eigveci = m_get(2*n_floquet+1, 2*n_floquet+1);
    eigvals = v_get(2*n_floquet+1);
    rhozero = zv_get(2*n_floquet+1);
    Uinv_rho = zv_get(2*n_floquet+1);
    sideb = v_get(2*n_floquet+1);
    v_zero(sideb);

    spc = v_get(NPOINTS);
    v_zero(spc);

    /* set up Fibonacci numbers for ZCW average */
    makefibon(fibon);

    if ((outp=fopen("spectrum.txt","wt")) == NULL)
    {
        printf("cannot open file.\n");
        exit(0);
    }
}

```

```

/*
 * calculate the powder average over the whole sphere
 * use Zaremba Conroy Wolfsberg grid - this code
 * derived from the SIMPSON program
 * Bak, Rasmussen and Neilsen, J. Mag. Res 147, 296-330 (2000)
 */
printf("Calculating %d angles\n", fibon[fibon_order]-1);
for (i=1; i<fibon[fibon_order]; i++)
{
    fac = i/(double)fibon[fibon_order];
    f0 = fac * 1.0;
    f1 = fac * (double)fibon[fibon_order-2];

    f0 -= floor(f0);
    f1 -= floor(f1);
    alpha = 2 * PI * f0; /* azimuthal */
    beta = PI * f1; /* polar */

    weight = (1.0/(double)fibon[fibon_order]) * sin(beta);;
    gen_freq(alpha, beta, n_floquet, theta,
             sigm11, sigma22, sigma33, omegar, nuzero, freqre, freqim,
             flmat_re, flmat_im, eigvecr, eigveci, eigvals, rhozero, Uinv_rho);
    accum(weight, omegar, n_floquet, centreband, eigvals, spc, rhozero, Uinv_rho, sideb);
}

/* print out the spectrum */
freq = START_FREQ;
for (j=0; j<NPOINTS; j++)
{
    fprintf(outp, "%10.2f %16.6f\n", freq, spc->ve[j]);
    freq -= DOMEQ/(2*PI);
}

fclose(outp);

for(i=0; i<2*n_floquet+1; i++)
{
    printf("%4d %16.4f\n", (i-n_floquet), 100*sideb->ve[i]/sideb->ve[n_floquet]);
}

printf("Done\n");
} /* end of main program */

/*****/

void gen_freq(double alpha, double beta, int n_floquet, double theta,
             double sigm11, double sigma22, double sigma33,
             double omegar, double nuzero, VEC *freqre, VEC *freqim,
             MAT *flmat_re, MAT *flmat_im, MAT *eigvecr, MAT *eigveci, VEC *eigvals,
             ZVEC *rhozero, ZVEC *Uinv_rho)
/*
 * set up and diagonalize the Floquet matrix
 * return eigenvalues and eigenvectors, then to
 * be used elsewhere for spectrum calculation
 */
{
    int i, j, m;
    int matz, ierr;

    double a1, b1, a2, b2;
    double sal, cal, sal2, cal2, sbt, cbt, sbt2, cbt2;
    double aa, bb;
    double coss, sins;
    double gamma, phi, sgam, cgam, x, y;
    double sigmaav;

    static VEC *wi = VNULL;
    static VEC *fv1 = VNULL, *fv2 = VNULL, *fv3 = VNULL;

```

```

static ZMAT *U = ZMNULL;
static ZMAT *Ud = ZMNULL;
static PERM *pivot = PNULL;

static ZVEC *sigma = ZVNULL;
static ZVEC *trsig = ZVNULL;
static ZVEC *trsig_r = ZVNULL;
static ZVEC *wksp = ZVNULL;
static ZVEC *Btr = ZVNULL;

/* allocate temporary storage */
sigma = zv_resize(sigma, 5);
trsig = zv_resize(trsig, 5);
trsig_r = zv_resize(trsig_r, 5);
wksp = zv_resize(wksp, 5);

Btr = zv_resize(Btr, 5);
zv_zero(Btr);

U = zm_resize(U, 2*n_floquet+1, 2*n_floquet+1);
Ud = zm_resize(Ud, 2*n_floquet+1, 2*n_floquet+1);

wi = v_resize(wi, 2*n_floquet+1);
fv1 = v_resize(fv1, 2*n_floquet+1);
fv2 = v_resize(fv2, 2*n_floquet+1);
fv3 = v_resize(fv3, 2*n_floquet+1);

/* express shift tensor as spherical tensor */
zv_zero(sigma);
/* zero order */
sigmaav = (sigma11+sigma22+sigma33)/3.0;
/* second order */
/* use the notation 2+0 to remind us the z component runs from -2 to +2 */
sigma->ve[2+0].re = sqrt(1.0/6.0) * (2*sigma33 - sigma11 - sigma22);
sigma->ve[2+2].re = (1.0/2.0) * (sigma11 - sigma22);
sigma->ve[2-2].re = (1.0/2.0) * (sigma11 - sigma22);

/* work out the trigonometry for each vertex */
cbr = cos(beta);          sbr = sin(beta);          /* polar */
cal = cos(alpha);        sal = sin(alpha);          /* azimuthal angle */

sal2 = sal*sal;          cal2 = 1.0 - sal2;
sbr2 = sbr*sbr;          cbr2 = 1.0 - sbr2;

/*
 * Rotate the magnetic field to the rotor angle (magic?)
 * since there is only Bz in the lab frame, we need only the
 * one row of the Wigner matrix
 */
Btr->ve[2+2].re = sqrt(3.0/8.0)*SQR(sin(theta));
Btr->ve[2+1].re = sqrt(3.0/8.0)*(sin(2.0*theta));
Btr->ve[2+0].re = 0.5 * (3.0*SQR(cos(theta))-1);
Btr->ve[2-1].re = -sqrt(3.0/8.0)*(sin(2.0*theta));
Btr->ve[2-2].re = sqrt(3.0/8.0)*SQR(sin(theta));

/* rotate CSA tensor into rotor frame */
/* first from some arbitrary frame, to show we can do it */
aa = 0.4;
coss = cos(aa);  sins = sin(aa);
mult_rz(coss, sins, sigma, wksp);
bb = 1.1;
coss = cos(bb);  sins = sin(bb);
mult_rz(coss, sins, wksp, trsig);
aa = 0.7;
coss = cos(aa);  sins = sin(aa);
mult_rz(coss, sins, trsig, sigma);

/* now for real */
mult_rz(cal, sal, sigma, wksp); /* around z */
mult_d2(cbr, sbr, wksp, trsig); /* around y */

```

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/*
 * Fourier components for Floquet
 * These are the analytical results, since this is a simple system
 * These are the terms that appear at the rotor frequency and
 * double the rotor frequency. Since the rotation with the gamma
 * angle corresponds to rotor rotation, these are just the
 * +1 and +2 spherical tensor components
 */
a1 = -trsig->ve[2+1].re; /* minus sign from contraction (-1)^m */
b1 = -trsig->ve[2+1].im;
a2 = trsig->ve[2+2].re;
b2 = trsig->ve[2+2].im;

#if H_AND_B
/*
 * These terms direct from Herzfeld and Berger
 * These are the sine and cosine terms - in Vega et al,
 * they use magnitude and phase, but the answer is the same
 */
a1 = sbt*cbt*(cal2*(sigma11-sigma33)+sal2*(sigma22-sigma33));
b1 = - sal*cal*sbt*(sigma11-sigma22);
a2 = 0.5*(cbt2*cal2-sal2)*(sigma11-sigma33) + 0.5*(cbt2*sal2-cal2)*(sigma22-sigma33);
b2 = - sal*cal*cbt*(sigma11-sigma22);
#endif

#if FFT
/*
 * Now get Fourier components the hard way.
 * explicitly do the gamma rotation and Fourier transform
 */

phi = 0.0;
for (i=0; i<NGAMMA; i++)
{
    gamma = i * (2.0 * PI)/((double) NGAMMA);
    gamma += phi;
    cgam = cos(gamma);    sgam = sin(gamma);
    mult_rz(cgam, sgam, trsig, trsig_r);
    freqre->ve[i] = 0.0;
    freqim->ve[i] = 0.0;
    for(j=-2; j<3; j++)
    {
        x = Btr->ve[2-j].re * trsig_r->ve[2+j].re;
        y = Btr->ve[2-j].re * trsig_r->ve[2+j].im;
        if (j%2) /* this is the (-1)^m factor */
        {
            x = -x;    y = -y;
        }
        freqre->ve[i] += x;
        freqim->ve[i] += y;
    }
}
/*
 * We have the frequency as a function of angle now,
 * do the fft, in place
 */
fft(freqre, freqim);

/* normalize the FT */
for (i=0; i<NGAMMA; i++)
{
    freqre->ve[i] = freqre->ve[i]/((double) NGAMMA);
    freqim->ve[i] = freqim->ve[i]/((double) NGAMMA);
}
#endif

/*
 * Make up the Floquet matrix
 * zero the Floquet matrix. Keep real and imaginary part separate
 * only because of the eigenvalue routine
 */

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m_zero(flmat_re);
m_zero(flmat_im);

/*
 * Put in diagonal (time independent) elements in Floquet matrix
 */
for (m=1; m<=n_floquet; m++)
{
    flmat_re->me[n_floquet+m][n_floquet+m] = nuzero * sigmaav + m * omegar;
    flmat_re->me[n_floquet-m][n_floquet-m] = nuzero * sigmaav - m * omegar;
}
flmat_re->me[n_floquet][n_floquet] = nuzero * sigmaav;

/* allow for non-magic angles */
for (m=0; m<2*n_floquet+1; m++)
{
    flmat_re->me[m][m] += nuzero * (sqrt(2.0/3.0)) * Btr->ve[2+0].re * trsig->ve[2+0].re;
}

/*
 * Put in the time-dependent terms, either from analytical terms or from numerical
 * Fourier analysis of explicit rotation about rotor axis
 */

#if ANALYTICAL
/* first off-diagonals */
/* sqrt(2.0)/3.0 comes from rotating static field into rotor frame */
for (m=0; m<2*n_floquet; m++)
{
    flmat_re->me[m][m+1] = nuzero * (sqrt(2.0/3.0)) * Btr->ve[2+1].re * a1;
    flmat_im->me[m][m+1] = -nuzero * (sqrt(2.0/3.0)) * Btr->ve[2+1].re * b1;
    flmat_re->me[m+1][m] = nuzero * (sqrt(2.0/3.0)) * Btr->ve[2+1].re * a1;
    flmat_im->me[m+1][m] = nuzero * (sqrt(2.0/3.0)) * Btr->ve[2+1].re * b1; /* c conj */
}

/* second off-diagonal */
for (m=0; m<2*n_floquet-1; m++)
{
    flmat_re->me[m][m+2] = nuzero * (sqrt(2.0/3.0)) * Btr->ve[2+2].re * a2;
    flmat_im->me[m][m+2] = -nuzero * (sqrt(2.0/3.0)) * Btr->ve[2+2].re * b2;
    flmat_re->me[m+2][m] = nuzero * (sqrt(2.0/3.0)) * Btr->ve[2+2].re * a2;
    flmat_im->me[m+2][m] = nuzero * (sqrt(2.0/3.0)) * Btr->ve[2+2].re * b2; /* c conj */
}
#endif

#if FFT
/* sqrt(2.0)/3.0 comes from rotating static field into rotor frame */
for (i=0; i<=NFOURIER; i++) /* loop over Fourier components */
{
    for (m=0; m<2*n_floquet+1-i; m++) /* fill in the Floquet matrix */
    {
        flmat_re->me[m][m+i] += nuzero * (sqrt(2.0/3.0)) * freqre->ve[i];
        flmat_im->me[m][m+i] += nuzero * (sqrt(2.0/3.0)) * freqim->ve[i];
        flmat_re->me[m+i][m] = flmat_re->me[m][m+i];
        flmat_im->me[m+i][m] = -flmat_im->me[m][m+i]; /* complex conj */
    }
}
#endif

/*
 * eispack routine for complex general matrix
 * the matrix is Hermitian, so this is overkill. However, we
 * will need it for exchange calculation.
 * If the shift tensor is cylindrical, the matrix is real
 * and a simple real eigenvalue routine will work
 */
matz = 1;
cg_(2*n_floquet+1, 2*n_floquet+1, flmat_re->me, flmat_im->me,
    eigvals->ve, wi->ve, matz, eigvecr->me, eigveci->me, fv1->ve, fv2->ve, fv3->ve, &ierr);

/* to work out intensities, allow for fact that matrix may not be hermitian */

```

```

for (i=0; i<2*n_floquet+1; i++)
{
    for (j=0; j<2*n_floquet+1; j++)
    {
        U->me[i][j].re = eigvecr->me[i][j];
        U->me[i][j].im = eigveci->me[i][j];
    }
}

/*
 * Calculate the spectrum as a sum of Lorentzian lines
 * This code for calculating the spectrum is from MEXICO
 * U is the matrix of eigenvectors of the Liouvillian
 * You need (Uinv * rho(0)) times (U * detector operator)
 * All the books say don't take inverses if you can avoid it
 * Treat uinv as a set of linear equations and solve by LU
 * decomposition. This gives the lines in the spectrum in
 * the eigenvector basis
 */
zv_zero(rhozero);
rhozero->ve[n_floquet].re = 1.0; /* middle is only nonzero element */
pivot = px_resize(pivot, 2*n_floquet+1);

zLUfactor(U, pivot); /* decompose */
zLUsolve(U, pivot, rhozero, Uinv_rho); /* and solve */

/* U * detector is trivial */
for (i=0; i<2*n_floquet+1; i++)
{
    rhozero->ve[i].re = eigvecr->me[n_floquet][i];
    rhozero->ve[i].im = eigveci->me[n_floquet][i];
}

} /* end of gen_freq */

/*****/

void accum(double weight, double omegar, int n_floquet, double centreband,
          VEC *eigvals, VEC *buffer,
          ZVEC *rhozero, ZVEC *Uinv_rho, VEC *sideb)
/* calculate the spectrum for each orientation and accumulate */
{
    int i, j;
    double intenst, harmonic, freq;
    double numer, dnumer, denom, omegav;

    Complex cplxfreq, numerator;
    Complex ints;

    /*
     * Calculate the intensities of each sideband, as well
     * as the complete spectrum
     * this is the square of the coefficient of the eigenvector
     */

    for (i=0; i<(2*n_floquet+1); i++)
    {
        /* accumulate the intensities of the sidebands */
        ints = zmlt(rhozero->ve[i], Uinv_rho->ve[i]);
        intenst = weight * ints.re;
        harmonic = (eigvals->ve[i]-centreband)/(omegar);
        for (j=0; j<2*n_floquet+1; j++)
        {
            if (fabs(harmonic + n_floquet - j) < 0.1)
                sideb->ve[j] += intenst;
        }

        /*
         * And calculate the lineshape for display
         * sweep over frequency for each complex
         * Lorentzian line

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*/
freq = START_FREQ;

cplxfreq.im = (2*PI*freq) - eigvals->ve[i];
cplxfreq.re = RELAX; /* arbitrary linewidth */
numerator.re = SCALE*intnst;
numerator.im = 0.0;
/* set up for efficient calculation of real part of (numerator/cplxfreq) */
numer = numerator.re * cplxfreq.re + numerator.im * cplxfreq.im;
dnumer = numerator.im * DOMEQ;
denom = SQR(cplxfreq.re);
omegav = cplxfreq.im;
for (j=0; j<NPOINTS; j++)
{
    buffer->ve[j] += (numer/(denom + SQR(omegav)));
    numer -= dnumer;
    omegav -= DOMEQ;
}
}
}

```

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/*****

```

```

void mult_d2(double cbt, double sbt, ZVEC *in, ZVEC *out)
/* apply the second-order Wigner matrix for rotation by angle beta around y */

```

```

{
    static ZMAT *d2 = ZMNULL;

    d2 = zm_resize(d2, 5, 5); /* second order Wigner matrix */
    zm_zero(d2);

    /* define Wigner elements */
    d2->me[2+2][2+2].re = 0.25 * SQR(1+cbt);
    d2->me[2+2][2+1].re = -0.5 * (1+cbt) * sbt;
    d2->me[2+2][2+0].re = sqrt(3.0/8.0) * SQR(sbt);
    d2->me[2+2][2-1].re = -0.5 * (1-cbt) * sbt;
    d2->me[2+2][2-2].re = 0.25 * SQR(1-cbt);

    d2->me[2+1][2+2].re = -d2->me[2+2][2+1].re;
    d2->me[2+1][2+1].re = 0.5 * (cbt-1) + SQR(cbt);
    d2->me[2+1][2+0].re = -sqrt(3.0/2.0) * sbt * cbt;
    d2->me[2+1][2-1].re = 0.5 * (cbt+1) - SQR(cbt);
    d2->me[2+1][2-2].re = d2->me[2+2][2-1].re;

    d2->me[2+0][2+2].re = d2->me[2+2][2+0].re;
    d2->me[2+0][2+1].re = -d2->me[2+1][2+0].re;
    d2->me[2+0][2+0].re = 0.5 * (3 * SQR(cbt) - 1);
    d2->me[2+0][2-1].re = d2->me[2+1][2+0].re;
    d2->me[2+0][2-2].re = d2->me[2+2][2+0].re;

    d2->me[2-1][2+2].re = -d2->me[2+2][2-1].re;
    d2->me[2-1][2+1].re = d2->me[2+1][2-1].re;
    d2->me[2-1][2+0].re = -d2->me[2+1][2+0].re;
    d2->me[2-1][2-1].re = d2->me[2+1][2+1].re;
    d2->me[2-1][2-2].re = d2->me[2+2][2+1].re;

    d2->me[2-2][2+2].re = d2->me[2+2][2-2].re;
    d2->me[2-2][2+1].re = -d2->me[2+2][2-1].re;
    d2->me[2-2][2+0].re = d2->me[2+2][2+0].re;
    d2->me[2-2][2-1].re = -d2->me[2+2][2+1].re;
    d2->me[2-2][2-2].re = d2->me[2+2][2+2].re;

    zmv_mlt(d2, in, out);
}

```

```

/*****

```

```

void mult_rz(double cal, double sal, ZVEC *in, ZVEC *out)
/* apply the second-order Wigner matrix for rotation by angle alpha around z */

```

```

{
    static ZMAT *rz = ZMNULL;

```



```

rz = zm_resize(rz, 5, 5);      /* rotation around z, for second order */
zm_zero(rz);

rz->me[2+2][2+2].re = 2 * SQR(cal) - 1;  /* cos(2 alpha) */
rz->me[2+2][2+2].im = 2 * sal * cal;    /* sin(2 alpha) */

rz->me[2+1][2+1].re = cal;
rz->me[2+1][2+1].im = sal;

rz->me[2+0][2+0].re = 1.0;

rz->me[2-1][2-1].re = cal;
rz->me[2-1][2-1].im = -sal;

rz->me[2-2][2-2].re = 2 * SQR(cal) - 1;
rz->me[2-2][2-2].im = -2 * sal * cal;

zmv_mlt(rz, in, out);
}

/*****

int makefibon(int *fibon)
/* set up vector of Fibonacci numbers */
{
    int i;

    fibon[0] = 0;
    fibon[1] = 1;

    for (i=2; i<MAX_FIBONACCI; i++)
        fibon[i] = fibon[i-1] + fibon[i-2];
}

*****/

```